

Goldbach's Conjecture

The Prussian mathematician Christian Goldbach noticed a pattern in the following sums:

$$\begin{array}{lll} 4 = 2+2, & 6 = 3+3, & 8 = 3+5 \\ 10 = 3+7 = 5+5 & 12 = 5+7, & 14 = 3+11=7+7, \\ 16 = 3+13 = 5+11, & 18 = 5+13 = 7+11, & 20 = 3+17 = 7+13 \end{array}$$

(Do you see a pattern here?) Based on his observations, Goldbach, in a letter to Euler in 1742, conjectured that every even integer > 2 can be expressed as the sum of two primes. Euler could not prove it, and his conjecture still remains an unsolved problem.

The famous English mathematician Godfrey H. Hardy (1877–1947) characterized Goldbach's conjecture as one of the most difficult unsolved problems in mathematics. However, Goldbach's conjecture was verified in 1998 by Joerg Richstein up to 4×10^4 .

In May 2000, Bloomsbury Publishing (United States) and Faber and Faber (United Kingdom) announced a million dollar prize to anyone who could provide a proof of Goldbach's conjecture by March 15, 2002. To date, it still remains a conjecture.

Ironically, Goldbach himself would not have been allowed to enter—Item 10 of the Terms and Conditions restricted the challenge to “legal residents of the United Kingdom and the United States aged 18 or over”!

Goldbach also conjectured in a letter to Euler of 1752 that every odd integer is the sum of a prime (which for Goldbach included the number 1) and double a square. Moritz Stern discovered in 1856 that 5777 and 5993 are exceptions.

Christian Goldbach (1690–1764) was born in Königsberg, Prussia. He studied medicine and mathematics at the University of Königsberg and became professor of mathematics at the Imperial Academy of Sciences in St. Petersburg in 1725. In 1728 he moved to Moscow to tutor Tsarevich Peter II and his cousin Anna of Courland. During 1729–1763, he corresponded with Euler on number theory. He returned to the Imperial Academy in 1732 when Peter's successor Anna moved the imperial court to St. Petersburg.



In 1742 Goldbach joined the Russian Ministry of Foreign Affairs, and later became privy councilor and established guidelines for the education of royal children.

He is also noted for his conjectures in number theory and work in analysis. Goldbach died in Moscow.

Excerpted from the following books

Thomas Koshy *Elementary Number Theory with Applications*, 2nd edition Academic Press, 2007.

David Wells *Prime Numbers The Most Mysterious in Math*, John Wiley and Sons, 2005