

Why complex number system is introduced?

Mathematics has its own language in which Alphabets are numbers, so number system is so important in mathematics. The prince of mathematics Gauss said that every polynomial has at least one root and that Polynomial has maximum roots exactly equal to its order or degree. Above statement is called the fundamental theorem of Algebra. When we consider the polynomial equation like as $x^2 + 1 = 0$ that implies $x^2 = -1$ but which is not possible in the real field because square of any real number is non-negative so real field fails to give the solution of this polynomial equation. But fundamental theorem of Algebra tells it has maximum two roots. To permit the solution of the equation $x^2 + 1 = 0$ and similar types, the set of complex numbers is introduced. Therefore complex number system includes real number system as a subset, so complex number system is the extended form of real number system that solved the above considered problem. By considering $i^2 = -1$ the problem $x^2 + 1 = 0$ provides two complex roots that covered the fundamental theorem of Algebra stated by great mathematician Gauss. It is notable that the mathematician Euler first use the symbol i for imaginary unit and its geometrical meaning in the complex plane is the point $(0,1)$. Solution of the above arises problem is as follows:

$$\begin{aligned}x^2 + 1 &= 0 & \Rightarrow x^2 &= -1 \\ \Rightarrow x^2 &= i^2 & [\because i^2 = -1] \\ \Rightarrow x &= \pm\sqrt{i^2} & \Rightarrow x &= \pm i\end{aligned}$$

Finally we conclude that the roots of the aroused problem are $x = i$ and $x = -i$.